Discussion #15

Interpretations

Topics

- Interpretations
- Closed World Assumption
- Interpretation Examples for Class Project

Interpretations Provide Meaning

- Consider the problem of giving meaning to the expression: sibling(x, Lynn) $\wedge$ married(x).
  - Can't just assign T or F to a predicate expression with variables
  - Truth depends on the values assigned to the variables
    - E.g. assign Zed to x; then if Zed is indeed Lynn's sibling and is married, we can say that this expression is true.
    - E.g. for $\exists$x(sibling(x, Lynn) $\wedge$ married(x)), we can look through the list of all possibilities (i.e. look through the domain) and see if at least one of them is a sibling of Lynn and is married; if so we can say that this expression is true.
- To provide an interpretation, we need
  - A domain that provides values for the arguments of the predicate
  - A way to determine the truth value of all predicates for each possible assignment of domain values to the variables

Interpretation

- An interpretation for an expression E
  - Specify a domain, D.
  - For each predicate of E, specify T or F for every possible substitution.
  - Select a value in D for each free variable, if any.
- Example: $\exists y P(x, y)$

$D = \{1, 2\}$

P(x, y)		= ?
1	1	T
1	2	F
2	1	F
2	2	F

$x = 1$: $\exists y P(x, y) = P(1, 1) \vee P(1, 2) = T \vee F = T$

$x = 2$: $\exists y P(x, y) = P(2, 1) \vee P(2, 2) = F \vee F = F$

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$x = 2$: $\exists y P(x, y) = P(2, 1) \vee P(2, 2) = T \vee F = T$

Observe that the truth of a statement depends on the interpretation.

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  - Select a value in D for each free variable, if any.
- Example: $\forall x \exists y P(x, y)$

$D = \{1, 2\}$

x	y	P(x, y) = ?
1	1	T
1	2	F
2	1	F
2	2	F

$$\forall x \exists y P(x, y) = \forall x (P(x, 1) \lor P(x, 2))$$
$$= (P(1, 1) \lor P(1, 2)) \land (P(2, 1) \lor P(2, 2))$$
$$= (T \lor F) \land (F \lor F) = T \land F = F$$

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  - Select a value in D for each free variable, if any.
- Example: $\forall x \exists y P(x, y)$

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x	y	P(x, y) = ?
1	1	T
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2	1	T
2	2	F

$$\forall x \exists y P(x, y) = \forall x (P(x, 1) \vee P(x, 2))$$
$$= (P(1, 1) \vee P(1, 2)) \wedge (P(2, 1) \vee P(2, 2))$$
$$= (T \vee F) \wedge (T \vee F) = T \wedge T = T$$

Commonly Understood Predicates

- Sometimes we already know the domain and the T/F value for every substitution.
- Example: $\exists y \forall x(x^2+1 > y)$
  - Assume we are discussing real numbers, so we know the domain.
  - Since we know the meaning of > and +, we know the T/F value for every substitution.
  - $\exists y \forall x(x^2+1 > y)$ = T (there is a y, e.g. 0 or 0.5 or …, such that for every x, $x^2+1 > y$)

Closed World Assumption

- With the closed world assumption, we only give the substitutions that evaluate to true — all others are assumed to be false.
- Let the domain, D = {1, 2}, then if we write:

P(x, y)	or	
1 1		P(1, 1)
1 2		P(1, 2)

- Then with the closed world assumption, this is simply shorthand for writing:

P(x, y) = ?	or	
1 1 T		P(1, 1) = T
2 1 F		P(2, 1) = F
1 2 T		P(1, 2) = T
2 2 F		P(2, 2) = F

Closed World Assumption Notes

- Our project uses the closed world assumption.
  - Only the substitutions that hold are given.
  - These are called facts.
  - Example, the facts in the Snoopy Database on the next slide
- Real-world databases use the closed world assumption — only "true" facts are stored.
- Contrary to the closed world assumption, the open world assumption says that if a fact is not stated, we do not know whether it is true or false.
  - The open world assumption is typical in everyday life.
  - It is harder to work with an open world assumption.

Schemes:

```
snap(S,N,A,P)
csg(C,S,G)
cp(C,Q)
cdh(C,D,H)
cr(C,R)
```

Facts:

```
snap('12345','C. Brown','12 Apple St.','555-1234').
snap('67890','L. Van Pelt','34 Pear Ave.','555-5678').
snap('22222','P. Patty','56 Grape Blvd.','555-9999').
snap('33333','Snoopy','12 Apple St.','555-1234').
csg('CS101','12345','A').
csg('CS101','67890','B').
csg('EE200','12345','C').
csg('EE200','22222','B+').
csg('EE200','33333','B').
csg('CS101','33333','A-').
csg('PH100','67890','C+').
cp('CS101','CS100').
cp('EE200','EE005').
cp('EE200','CS100').
cp('CS120','CS101').
cp('CS121','CS120').
cp('CS205','CS101').
cp('CS206','CS121').
cp('CS206','CS205').
cdh('CS101','M','9AM').
cdh('CS101','W','9AM').
cdh('CS101','F','9AM').
cdh('EE200','Tu','10AM').
cdh('EE200','W','1PM').
cdh('EE200','Th','10AM').
cdh('PH100','Tu','11AM').
cr('CS101','Turing Aud.').
cr('EE200','25 Ohm Hall').
cr('PH100','Newton Lab.').
```

Rules:

```
WhoGradeCourse(N,G,C):-csg(C,S,G),snap(S,N,A,P).
before(C1,C2):-cp(C2,C1).
before(C1,C2):-cp(C3,C1),before(C3,C2).
```

Queries:

```
WhoGradeCourse('Snoopy',G,C)?
WhoGradeCourse(N,'A','CS100')?
WhoGradeCourse(N,'A',C)?
before('CS100','CS206')?
before('CS100',X)?
```

Example #1 (Class Project)

- Query: What are the prerequisites of EE200?
- Translated to predicate logic, we are asking for:

  cp('EE200', x)

  where: cp(course, prerequisite)
- We need to find the substitutions for the free variable x, if any, that make this true.
- Interpretation for the project:
  - Domain = all constant strings in the Facts
  - Closed world assumption holds (if stated as a fact, then T; otherwise, F).

Example #1 (continued…)

- Check all substitutions for cp('EE200', x) from the domain for x:

cp('EE200','10AM') = F
cp('EE200','11AM') = F
cp('EE200','12 Apple St.') = F
…
cp('EE200','CS100') = T
…
cp('EE200','EE005') = T
…

x = 'CS100'
x = 'EE005'

cp Facts:

cp('CS101','CS100').
cp('EE200','EE005').
cp('EE200','CS100').
cp('CS120','CS101').
cp('CS121','CS120').
cp('CS205','CS101').
cp('CS206','CS121').
cp('CS206','CS205').

- Also,
  - $\exists$x cp('EE200', x) = T
  - $\forall$x cp('EE200', x) = F
  - $\exists$x cp('CS100', x) = F

Example #2 (Class Project)

- Query: Where am I likely to find Charlie Brown ('12345') on Wednesday ('W') at 1 PM ('1PM')?
- Translated to predicate logic, we are asking for:

$\exists x \exists z(csg(x,'12345',z) \land cr(x,r) \land cdh(x,'W','1PM'))$

where:
csg(course, studentID, grade)
cr(course, room)
cdh(course, day, hour)
r is a free variable.

Example #2 (continued…)

Check substitutions for all combinations of values from the domain for x, z, and r:

…

csg('10AM','12345','10AM') ∧ cr('10AM','10AM') ∧ cdh('10AM','W','1PM') = F
csg('10AM','12345','10AM') ∧ cr('10AM','11AM') ∧ cdh('10AM','W','1PM') = F
csg('10AM','12345','10AM') ∧ cr('10AM','12 Apple St.') ∧ cdh('10AM','W','1PM') = F
…
csg('10AM','12345','11AM') ∧ cr('10AM','10AM') ∧ cdh('10AM','W','1PM') = F
csg('10AM','12345','11AM') ∧ cr('10AM','11AM') ∧ cdh('10AM','W','1PM') = F
csg('10AM','12345','11AM') ∧ cr('10AM','12 Apple St.') ∧ cdh('10AM','W','1PM') = F
…
csg('11AM','12345','10AM') ∧ cr('11AM','10AM') ∧ cdh('11AM','W','1PM') = F
csg('11AM','12345','10AM') ∧ cr('11AM','11AM') ∧ cdh('11AM','W','1PM') = F
csg('11AM','12345','10AM') ∧ cr('11AM','12 Apple St.') ∧ cdh('11AM','W','1PM') = F
…
csg('11AM','12345','11AM') ∧ cr('11AM','10AM') ∧ cdh('11AM','W','1PM') = F
csg('11AM','12345','11AM') ∧ cr('11AM','11AM') ∧ cdh('11AM','W','1PM') = F
csg('11AM','12345','11AM') ∧ cr('11AM','12 Apple St.') ∧ cdh('11AM','W','1PM') = F
…

Example #2 (continued…)

- Eventually, we check the substitution x = **'EE220'**, z = **'C'**, and r = **'25 Ohm Hall'**.

  csg('EE220','12345','C') ∧ cr('EE220','25 Ohm Hall') ∧ cdh('EE220','W','1PM') = T

- Thus, Charlie Brown is likely to be in 25 Ohm Hall on Wednesday at 1 PM.

csg, cdh, and cr Facts:

csg('CS101','12345','A').
csg('CS101','67890','B').
csg('EE200','12345','C').
csg('EE200','22222','B+').
csg('EE200','33333','B').
csg('CS101','33333','A-').
csg('PH100','67890','C+').
cdh('CS101','M','9AM').
cdh('CS101','W','9AM').
cdh('CS101','F','9AM').
cdh('EE200','Tu','10AM').
cdh('EE200','W','1PM').
cdh('EE200','Th','10AM').
cdh('PH100','Tu','11AM').
cr('CS101','Turing Aud.').
cr('EE200','25 Ohm Hall').
cr('PH100','Newton Lab.').